Polynomials

In MatDeck we use symbolics as a natural way of working with math objects. As a result, to define polynomials and work with them is a very simple task.

A polynomial is a function in the form

$$f(x) = a_n \cdot x^n + a_{n-1} \cdot x^{n-1} + ... + a_2 \cdot x^2 + a_1 \cdot x + a_0$$

When we have to judge the degree of a polynomial we use the highest power of x in its expression, n in this case.Constant polynomials are polynomials with a degree of zero, linear polynomials have a degree of one, quadratic polynomials have a degree of zero, ...

To use polynomials and do basic algebraic operations with them, you can either use functions designed for polynomial or simply do calculations as you would write it traditionally on paper. We will define two variables and place polynomials within them,to illustrate these properties.

Polynomials	$a := x^5 - 1$	$b := x - 1$
Operation	Using function	Without function
Addition	$\text{poladd}(a, b) = x^5 + x - 2$	$a + b = x^5 - 2 + x$
Subtraction	$\text{polsub}(a, b) = x^5 - x$	$a - b = x^5 - x$
Multiplication	$\text{polmul}(a, b) = x^6 - x^5 - x + 1$	$a \cdot b = x^6 - x^5 - x + 1$
Division	$\text{poldiv}(a, b) = \begin{bmatrix} x^4 + x^3 + x^2 + x + 1 & 0 \end{bmatrix}$	$a / b = x^4 + x^3 + x^2 + x + 1$

When we do polynomial division thats leaves a remainder, the resulting vector of the function **poldiv** will contain the remainder in the second position. On the other hand, normally division in this case will produce a fraction.

Polynomials	$a1 := x^5 - 1$	$b1 := x - 2$
Operation	Using function	Without function
Division	$\text{poldiv}(a1, b1) = \begin{bmatrix} x^4 + 2x^3 + 4x^2 + 8x + 16 & 31 \end{bmatrix}$	$a1 / b1 = \dfrac{x^5 - 1}{x - 2}$

There are several very useful functions from the polynomial group that we have implemented. For example, you can find the value of a polynomial for a specific value using the function, **polvalue**. Let's find out the value of polynomial $2x^4-3x^2+5x-6$ in the point 2.5

$$\text{polvalue}\left(2\,x^4-3\,x^2+5\,x-6\;,\;2.5\right)=65.875$$

To find the roots of a polynomial use the function, **polroots.** This function calculates both real and complex roots and returns them in a vector.

$$\text{polroots}\left(2\,x^4-3\,x^2+5\,x-6\right)=\begin{bmatrix}0.346-1.084i\\0.346+1.084i\\-1.907+0i\\1.215+0i\end{bmatrix}$$

There is an opposite function **roots2pol** that can return a polynomial based on the given roots vector. The argument vector can contain both real or complex roots

$$\text{roots2pol}\left(\begin{bmatrix}1 & -1 & 0 & 2\end{bmatrix},x\right)=x^4-2\,x^3-x^2+2\,x$$

$$\text{roots2pol}\left(\begin{bmatrix}2i & -1i & 1i\end{bmatrix},t\right)=(1+0i)\,t^3+(0-2i)\,t^2+(1+0i)\,t+0-2i$$

There is also a function that can extract the coefficients of the polynomial and return them as a vector called **pol2coef** and a function that can create a polynomial base on the given coefficients called **coef2pol**. Coefficients can be real or complex numbers in both functions and you can control the symbol of the polynomial that you want to create within the **coef2pol** function from it's second argument.

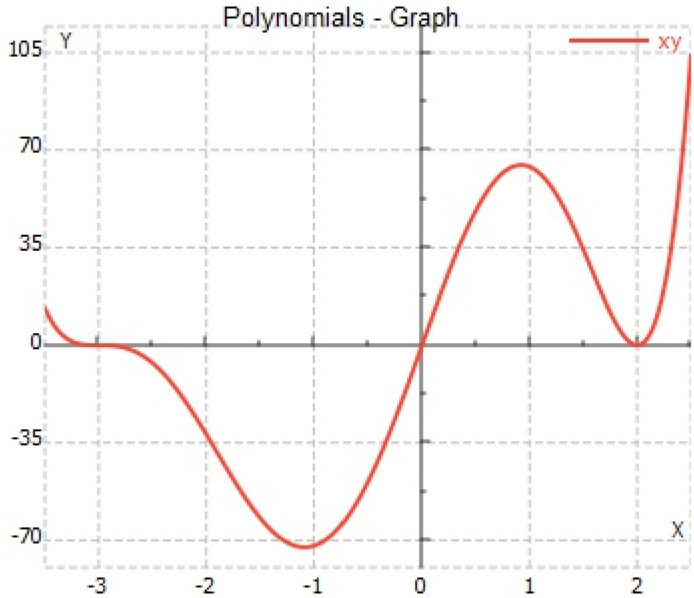
$$\text{pol2coef}\left(-2.1\cdot x^3+3\cdot x^2-6\,x+2\right)=\begin{bmatrix}-2.1 & 3 & -6 & 2\end{bmatrix}$$

$$\text{coef2pol}\left(\begin{bmatrix}-2.1 & 3 & -6 & 2\end{bmatrix},t\right)=-2.1\,t^3+3\,t^2-6\,t+2$$

We can easily join all the functions properties that we have described above to examine the given polynomial, to visualize it, find the polynomial roots, to find the value of a polynomial (function) at a certain point. Use the function **curve2d** to visualize the polynomial and create a complimentary graph.

Below, we will plot the polynomial function $x^6 + 5\,x^5 - 5\,x^4 -45\,x^3 +108\,x$, find the function roots and factorize the polynomial to make finding the roots more evident.

$$xy := \text{curve2d}\left(x^6 + 5x^5 - 5x^4 - 45x^3 + 108x, -3.5, 2.5, 1000\right)$$



$$\text{polroots}\left(x^6 + 5x^5 - 5x^4 - 45x^3 + 108x\right) = \begin{bmatrix} 2.000 + 0i \\ -2.998 - 0.003i \\ -2.998 + 0.003i \\ -3.004 + 0i \\ 2 + 0i \\ 0 + 0i \end{bmatrix}$$

If we write this polynomial in a different form and factorize it, it's roots will become evident so we can control the above result.

$$x^6 + 5x^5 - 5x^4 - 45x^3 + 108x = x \cdot (x-2)^2 \cdot (x-3)^3$$