System of linear equations

A system of linear equations is a system composed of a minimum of two linear equations with the same set of variables. To find the solution to a linear equations system we have to find the values of the variables that will satisfy all the equations simultaneously.

A system of n linear equations with n independent variables (x_1, x_2,..., x_n) in general form can be presented

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + \ldots + a_{1n} \cdot x_n = b_1$$
$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + \ldots + a_{2n} \cdot x_n = b_2$$
$$\ldots.$$
$$a_{n1} \cdot x_1 + a_{n2} \cdot x_2 + \ldots + a_{nn} \cdot x_n = b_n$$

If $b_1 = b_2 = \ldots = b_n$, for the above linear system, it can be said it is a homogenous system. Ordered n numbers (ε_1, ε_2,..., ε_n) are the solutions of the linear equation system if every equation in the system for $x_k = \varepsilon_k$ (k = 1, 2,..., n) comes down to identity. For systems that satisfy this condition, we can say it is a solvable system.

The general form of linear equation system can be presented in several different ways. One of them is Vector equation form.

$$x_1 \cdot \begin{bmatrix} a_{11} \\ a_{21} \\ \ldots \\ a_{n1} \end{bmatrix} + x_2 \cdot \begin{bmatrix} a_{12} \\ a_{22} \\ \ldots \\ a_{n2} \end{bmatrix} + \ldots + x_n \cdot \begin{bmatrix} a_{1n} \\ a_{2n} \\ \ldots \\ a_{nn} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{bmatrix}$$

Another common form of writing a system of linear equations is Matrix equation form.

$$\begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_m \end{bmatrix}$$

A - m x n matrix   X - column vector   b - column vector

In MatDeck we have implemented the functions **linsolvesys** - Linear system equation solver and **nonlinsolvesys** - Nonlinear system equation solver. Both of them are capable of solving solvable systems. Linear systems are solved in matrix forms and nonlinear systems are solved using numerical methods.

To solve linear equation systems using the linsolvesys function, insert the vector as the first function argument. Next, place a Equation object from the Math tab as the vector elements and start inserting the equations. We will show you several examples of linear equations system solving.

$$\text{linsolvesys}\left(\begin{bmatrix} 2x+3y-5z==-7 \\ -3x+2y+z==-9 \\ 4x-y+2z==17 \end{bmatrix}, \begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)=\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

$$\text{linsolvesys}\left(\begin{bmatrix} 2x+3y-5z==0 \\ -3x+2y+z==0 \\ 4x-y+2z==0 \end{bmatrix}, \begin{bmatrix} x \\ y \\ z \end{bmatrix}\right)=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{linsolvesys}\left(\begin{bmatrix} x1+x2+x3==3 \\ x2+x3+x4==4 \\ x3+x4+x5==5 \\ x4+x5+x6==6 \\ x5+x6+x7==7 \\ x6+x7+x1==8 \\ x7+x1+x2==9 \end{bmatrix}, \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \\ x6 \\ x7 \end{bmatrix}\right)=\begin{bmatrix} 3 \\ 1 \\ -1 \\ 4 \\ 2 \\ 3.331e-16 \\ 5 \end{bmatrix}$$

Syntax of using the function **nonlinsolvesys** is the same as for **linsolvesys**, and the differences are that you can use nonlinear equations in the system and you have to define the starting point for variables. If the system has several solutions, this starting point will localize the solution and the function will return the nearest solution. The function **nonlinsolvesys** can only calculate real solutions.

$$\text{nonlinsolvesys}\left(\begin{bmatrix} \sqrt[3]{x\cdot y}+\sqrt[2]{z}==4 \\ x^2+y^2+z^2==81 \\ \sqrt[2]{x}+y\cdot z==33 \end{bmatrix}, \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \begin{bmatrix} 2 \\ 10 \\ 5 \end{bmatrix}\right)=\begin{bmatrix} 1 \\ 8 \\ 4 \end{bmatrix}$$

To illustrate how the inserted starting point affects the final solution of a function, lets look at the following example.

$$\text{nonlinsolvesys}\left(\begin{bmatrix} x^4 + y^2 == 17 \\ x^2 + y^2 == 5 \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\text{nonlinsolvesys}\left(\begin{bmatrix} x^4 + y^2 == 17 \\ x^2 + y^2 == 5 \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\text{nonlinsolvesys}\left(\begin{bmatrix} x^4 + y^2 == 17 \\ x^2 + y^2 == 5 \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

As you can see, for three different starting point, we have calculated three different solutions of the inserted system.