

## Root Locus 2

Sketch the root locus of

$$KGH = \frac{64 k}{s (s+4) \cdot (s+16)} \quad (1)$$

### Solution:

In the equation above there are no zeros. The poles are at  $s = 0$ ,  $s = -4$ ,  $s = -16$ . Branches of loci start at open-loop poles and terminate at open-loop zeros at infinity.

The root locus exists on the real axis between  $s = 0$  and  $s = -4$ ,  $s = -16$  and  $s = -\infty$ . The root locus exists on the same real axis when an odd number of poles and zeros are found to the right of the point.

The asymmetrical angles are

$$\alpha = \frac{(2k+1) \cdot 180^\circ}{\Sigma P - \Sigma Z}$$

$$\alpha := \frac{(2k+1) \cdot 180}{3}$$

$$\alpha = 120k + 60 \quad (2)$$

$$k := 0$$

$$\text{I} \quad \alpha := \frac{(2k+1) \cdot 180}{3}$$

$$\alpha = 60$$

$$k := 1$$

$$\text{II} \quad \alpha := \frac{(2k+1) \cdot 180}{3}$$

$$\alpha = 180$$

$$k := 2$$

$$\text{III} \quad \alpha := \frac{(2k+1) \cdot 180}{3}$$

$$\alpha = 300$$

The center of gravity is given by

$$\Sigma Z := 0$$

$$\Sigma P := 3$$

$$CG = \frac{\Sigma P \text{ values} - \Sigma Z \text{ values}}{\Sigma P - \Sigma Z} \quad (3)$$

$$CG := \frac{-16 - 4 - 0 + 0}{\Sigma P - \Sigma Z}$$

$$CG = -6.667$$

The center of gravity gives the starting point for the asymptotic lines.

The breakaway point is given by

$$\frac{1}{S_b} = \frac{1}{16 - S_b} + \frac{1}{4 - S_b} \quad (4)$$

or

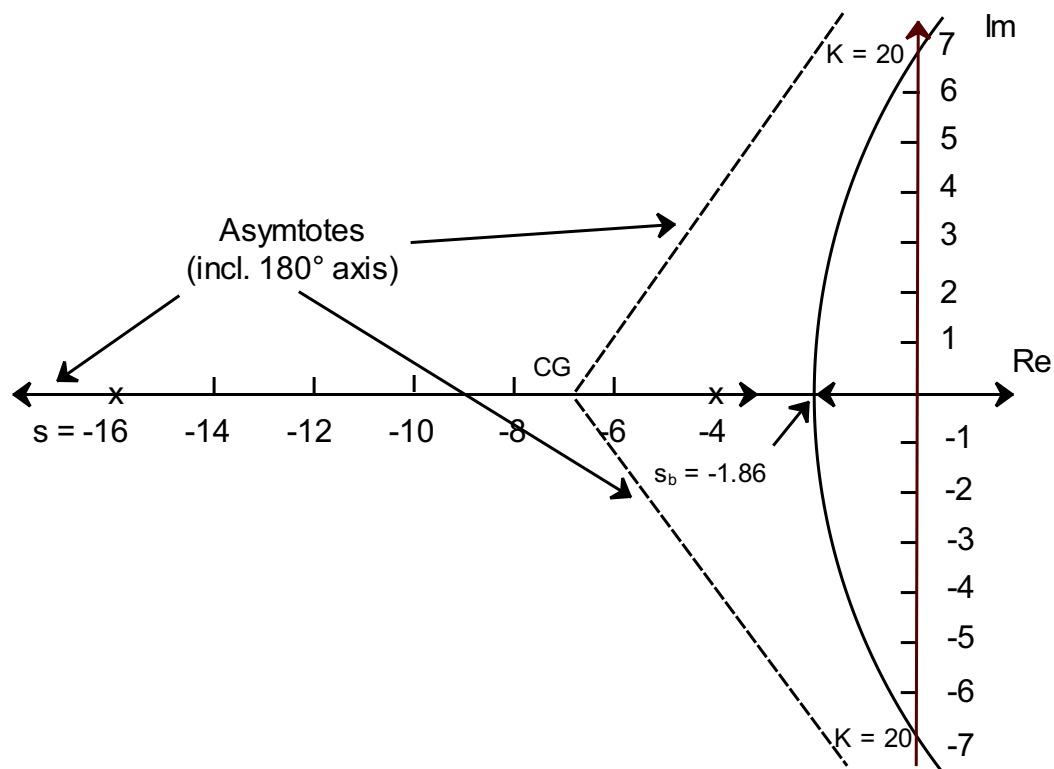
$$(16 - S_b) \cdot (4 - S_b) = S_b \cdot (4 - S_b) + S_b \cdot (16 - S_b)$$

The approximate value of  $S_b$  is

$$\text{nonlinsolve}((16 - S_b) \cdot (4 - S_b) == S_b \cdot (4 - S_b) + S_b \cdot (16 - S_b), S_b) = [1.859 \quad 11.474]$$

$$S_b \approx -1.86$$

The breakaway angle from the real axis is  $\pm 90^\circ$ .



The maximum value of K for which the system is stable is found by substituting  $s = j\varphi$ ,

$$KGH(j\varphi) = \frac{64 K}{j\varphi (j\varphi + 4) \cdot (j\varphi + 16)} \quad (5)$$

Setting  $KGH(j\varphi) = -1$  we have

$$\frac{64 K}{j\varphi (j\varphi + 4) \cdot (j\varphi + 16)} = -1 \quad (6)$$

Solving for K

$$K = \frac{20\varphi^2 + j\varphi(\varphi^2 - 64)}{64} \quad (7)$$

For K to be a real number  $\varphi^2 - 64$  must be equal to zero.

$$\varphi^2 = 64 = \left[ \varphi \begin{bmatrix} -8 & 8 \end{bmatrix} \right]$$

From equation (7), for  $\varphi = \pm 8$ , we get K

$$\varphi := 8$$

$$K := \frac{20 \cdot \varphi^2 + j\varphi(\varphi^2 - 64)}{64}$$

$$K = 20$$

The root locus of the system is shown in the figure below.

$$KGH = \frac{64 K}{s(s+4) \cdot (s+16)}$$