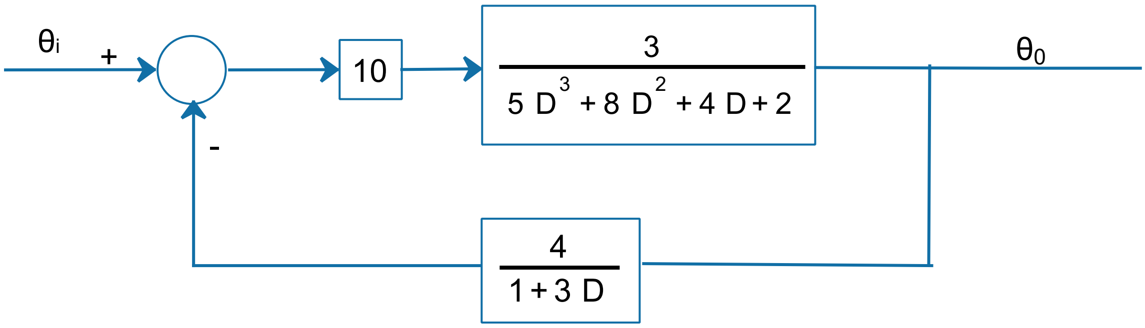
System stability

Using the Routh Chriterion, determine the stability of the system for the block diagram below.



Solution:

We will find the characteristic equation of the system

$$T := \frac{\theta_0}{\theta_i}$$

$$T := \frac{10 \cdot \frac{3}{5 D^3 + 8 D^2 + 4 D + 2}}{1 + \frac{4 \cdot 10 \cdot 3}{(1 + 3 D) \cdot \left(5 D^3 + 8 D^2 + 4 D + 2\right)}}$$

$$T := \frac{30 (1 + 3 D)}{(1 + 3 D) \cdot \left(5 D^3 + 8 D^2 + 4 D + 2\right) + 4 \cdot 10 \cdot 3}$$

The characteristic equation is

$$Chpol := (1 + 3 D) \cdot \left(5 D^3 + 8 D^2 + 4 D + 2\right) + 4 \cdot 10 \cdot 3$$

$$Chpol = 15 D^4 + 29 D^3 + 20 D^2 + 10 D + 122$$

The Routh array is

$$a := \begin{bmatrix} D^4 & 15 & 20 & 122 \\ D^3 & 29 & 10 & 0 \\ D^2 & A_1 & A_2 & 0 \\ D^1 & B_1 & B_2 & 0 \\ D^0 & C_1 & 0 & 0 \end{bmatrix}$$

$$A_1 := \frac{-1 \cdot \text{mat determinant}(\text{subset}(a, 0, 1, 1, 2))}{\text{value at}(a, 1, 1)}$$

$$A_1 = 14.828$$

$$A_2 := \text{value at}(a, 0, 3)$$

$$A_2 = 122$$

$$a := \begin{bmatrix} D^4 & 15 & 20 & 122 \\ D^3 & 29 & 10 & 0 \\ D^2 & A_1 & A_2 & 0 \\ D^1 & B_1 & B_2 & 0 \\ D^0 & C_1 & 0 & 0 \end{bmatrix}$$

$$B_1 := \frac{-1 \ \text{mat determinant}(\text{subset}(a, 1, 1, 2, 2))}{A_1}$$

$$B_1 = -228.609$$

As you can see, we have a sign change in the second matrix column so the system is not stable.