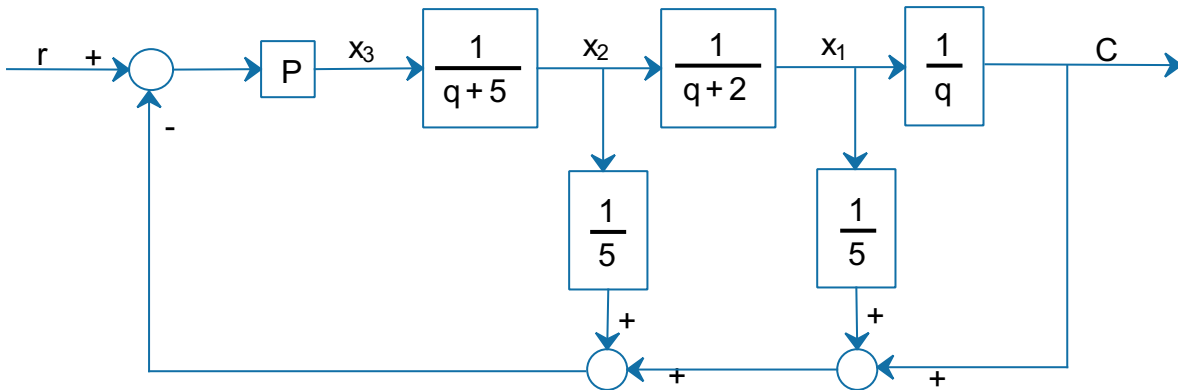


System stability

Examine the stability of the system shown in the picture below



Solution:

We shall first find the transfer function and the characteristic equation of the system.

$$c := \frac{P}{(q+5) \cdot (q+2) \cdot q} \cdot \left(r - \left(c + \frac{1}{5} \cdot x_1 + \frac{1}{5} \cdot x_2 \right) \right)$$

$$x_1 = cs$$

$$x_2 / (q+2) = x_1$$

$$x_2 = x_1 (q+2) = cs (q+2)$$

$$M := \frac{c}{r}$$

$$M := \frac{P}{q(q+5) \cdot (q+2) + \frac{1}{5} \cdot P (q^2 + 3q + 5)}$$

The characteristic equation of the system is

$$D_q := q(q+5) \cdot (q+2) + \frac{1}{5} \cdot P (q^2 + 3q + 5)$$

$$D_q = q^3 + 7q^2 + 10q + 0.2Pq^2 + 0.6Pq + P$$

Therefore the Routh array is:

$$\mathbf{a} := \begin{bmatrix} q^3 & 1 & 10+0.6 P \\ q^2 & 7+0.2 P & P \\ q^1 & B & 0 \\ q^0 & P & 0 \end{bmatrix}$$

$$\mathbf{B} := \frac{-1 \text{ mat determinant}(\text{subset}(\mathbf{a}, 0, 1, 1, 2))}{\text{value at}(\mathbf{a}, 1, 1)}$$

$$\mathbf{B} = \frac{0.1 P^2 + 5.2 P + 70}{0.2 P + 7}$$

For the system to be stable all the elements of the second column must to be positive.

$$7 + 0.2 P > 0 = [P \ (-35, \text{inf})]$$

$$\mathbf{B} > 0 = [P \ (-35, \text{inf})]$$

$$P > 0 = [P \ (0, \text{inf})]$$

When $P > 0$ all the elements of the array are positive and the system is stable.