Differential equations in MatDeck

In the world that surrounds us, there are lots of examples of the application of differential equations. In fact most physical models are nonlinear, it's hard to find a physical system that is not nonlinear and that can't be presented with differential equations and systems. A few examples where differential systems are used are:

- Kinetics of chemical reactions and most systems in general chemical engineering
- Electric AC motors (induction, permanent-magnet synchronous, synchronous reluctance,...)
- In Hooke's Law for modeling the motion of a spring or in representing models for population growth and money flow/circulation
- Dynamics of aircraft (nonlinear dependences on speed, angles, altitudes,...), helicopters, satellites, most systems in aerospace engineering
- In most robotic systems
- In medicine for cancer growth modeling
- In chemistry for modeling chemical reactions and to compute radioactive half life
- In physics to describe the motion of waves, pendulums or chaotic systems
- In physics with Newton's Second Law of Motion and the Law of Cooling

MatDeck has implemented functions for differential equations and systems solving, that uses numerical methods to calculate solutions and prepare this solutions for easy and instant visualization.
Functions that are designed for problems that uses differentiation are **desolve** and **desolvesys**.

Function **desolve** is a function design for solving of the differential equations and it is designed to accept following arguments:

- First argument - Equation we are solving (insert equation object and define equation); To insert derivatives and to create a preferred order use Shift + 6 key combination to create a power node; The order of the equation can be apostrophe (for orders from 1 to 3) or simple derivative number inside the brackets signs;
- Second argument - Starting value for independent variable;
- Third argument - Ending value for independent variable;
- Fourth argument - Number of steps for independent variable;
- Fifth argument - Number or vector (type of this argument depends of the order of the equation from the first argument, if it is the first order equation this argument is a number and if it is higher order equation this argument have to be a vector which size is equal to the order of the the equation) that presents the initial condition of the unknown function.

Examples of differential equation creation and solving

$$\text{desolve}\left(y' == e^{-y} \cdot (2\,x - 4),\ 5\ ,\ 5.5\ ,\ 3\ ,\ 0 \right) = \begin{bmatrix} 5 & 0 \\ 5.25 & 0.948 \\ 5.5 & 1.451 \end{bmatrix}$$

$$\text{desolve}\left(y'' == e^{-y} \cdot (2\,x - 4),\ 5\ ,\ 5.5\ ,\ 3\ ,\ \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \right) = \begin{bmatrix} 5 & 0 \\ 5.25 & 0.234 \\ 5.5 & 0.783 \end{bmatrix}$$

$$\text{desolve}\left(y^{(2)} == e^{-y} \cdot (2x-4), 5, 5.5, 3, \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} \right) = \begin{bmatrix} 5 & 0 \\ 5.25 & 0.234 \\ 5.5 & 0.783 \end{bmatrix}$$

$$\text{desolve}\left(y^{(4)} == 0.85 \cdot y, 2, 2.2, 4, \begin{bmatrix} 19 \\ 15 \\ 13 \\ 12 \end{bmatrix} \right) = \begin{bmatrix} 2 & 19 \\ 2.067 & 20.029 \\ 2.133 & 21.121 \\ 2.200 & 22.277 \end{bmatrix}$$

Function **desolvesys** is function for solving of systems of differential equations and it is designed to accept the following arguments:

- First argument - Vector of equations (insert vector of preferred size, create an equation objects as a elements of vector and define equations); Rules for equations defining are the same as for the first argument of the function desolve we have already explained earlier in this document;
- Second argument - Starting value for independent variable;
- Third argument - Ending value for independent variable;
- Fourth argument - Number of steps for independent variable;
- Fifth argument - Vector of initial conditions of the unknown function (vector size have to be the same as the size of the vector of equations from the first argument, each element of this vector corresponds to the equation from the first argument at the same position)

Examples of differential equations system creation and solving

$$\text{desolvesys}\left(\begin{bmatrix} x' == 2x-y-z \\ y' == 2x-y-2z \\ z' == -x+y+2z \end{bmatrix}, 0, 2, 3, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right) = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & -7.958 & -15.917 & 18.792 \\ 2 & -50.443 & -100.885 & 79.783 \end{bmatrix}$$

$$\text{desolvesys}\left(\begin{bmatrix} x' == 2x+y-1 \\ y' == -x+2y+8 \end{bmatrix}, 3, 3.1, 5, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 3 & 1 & 1 \\ 3.025 & 1.054 & 1.230 \\ 3.05 & 1.117 & 1.470 \\ 3.075 & 1.19 & 1.721 \\ 3.1 & 1.272 & 1.983 \end{bmatrix}$$

We will show a few examples of usage of differential equations and system in some common situations.

<u>Example 1</u>

Visualize the rate at which a chemical reaction of the decomposition of nitrogen dioxide occurs.

$$2\ NO_2 \rightarrow 2\ NO + O_2$$

We will use the usual chemistry notation, where $[NO_2]$ denotes the concentration of NO_2.
The reaction requires two molecules of NO_2, so the rate at which the reaction occurs is proportional to the square of the concentration of NO_2.

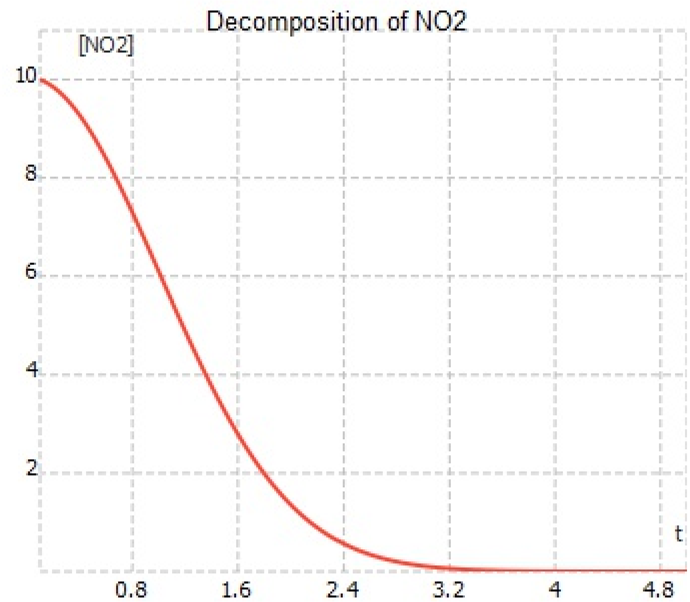
$$\frac{d\,[NO_2]}{dt} = -k\,[NO_2]^2$$

For better visualization we will use a single letter for concentration of NO_2, for example G. The above exponential equation transforms to

$$\frac{dH}{dt} = -k \cdot G^2$$

Solving of this equation will represent the trend of decomposition of NO_2 (G) in time (t).

$$a := \text{desolve}\left(G' == -k \cdot G\ ,\ 0.1\ ,\ 5\ ,\ 1000\ ,\ 10 \right)$$



The calculated graph on the picture above shows us that the decomposition decreases very quickly at first, but then very slowly afterwards.

Example 2

A manufacturer has a marketing policy based upon the price x(t) of its product. The goal is to set price x(t) dynamically to reflect demand for the product.
The production P(t) and the sales S(t) are given in terms of the price x(t) and the change of price x'(t) by the equation

$$P(t) = 4 - \frac{3}{4} \cdot x(t) - 8 \cdot x'(t) \qquad \text{(Production)}$$
$$S(t) = 15 - 4 \cdot x(t) - 2 \cdot x'(t) \qquad \text{(Sales)}$$

The differential equations for the price x(t) and the inventory level I(t) are following (the desired inventory level is 50). The initial values are x(0) = 10 and I(0) = 7.

$$x'(t) = k \cdot I(t) - k \cdot 50$$
$$I'(t) = \frac{13}{4} \cdot x(t) - 6 \cdot k \cdot I(t) + 300 \cdot k - 11$$

$$b := \text{desolvesys}\left(\begin{bmatrix} x' == k \cdot y - 50 \cdot k \\ y' == \frac{13}{4} \cdot x - 6\, k \cdot y + 300 \cdot k - 11 \end{bmatrix}, 1, 15, 1000, \begin{bmatrix} 10 \\ 7 \end{bmatrix} \right)$$

To find the the price x(t) that suites to the desired level of $I_0 = 50$, we will find the value from column that represents level values that is the closest to desired level. We will extract columns that contains values for price and level, and place them in variable c.

$$c := \text{subset}(b, 35, 1, 999, 2)$$

Extraction of columns for price and inventory level

$$\text{maxpoint}(c) = \begin{bmatrix} 3.291 & 49.977 \end{bmatrix}$$

Find the price that correspond to inventory level that is closest to 50

So the best forecast of price is x(t) = 3.29 at inventory level I(t) = 50.