

Two-Port Networks and Filters

Filters

By combining resistors, capacitors, and inductors in special ways, you can design networks that are capable of passing certain frequencies of signals while rejecting others. This section examines four basic kinds of filters: low-pass, high-pass, band-pass and notch-pass.

Low-Pass Filters

The simple RC filter shown in Figure 1. acts as low-pass filter - it passes low frequencies but reject high frequencies.

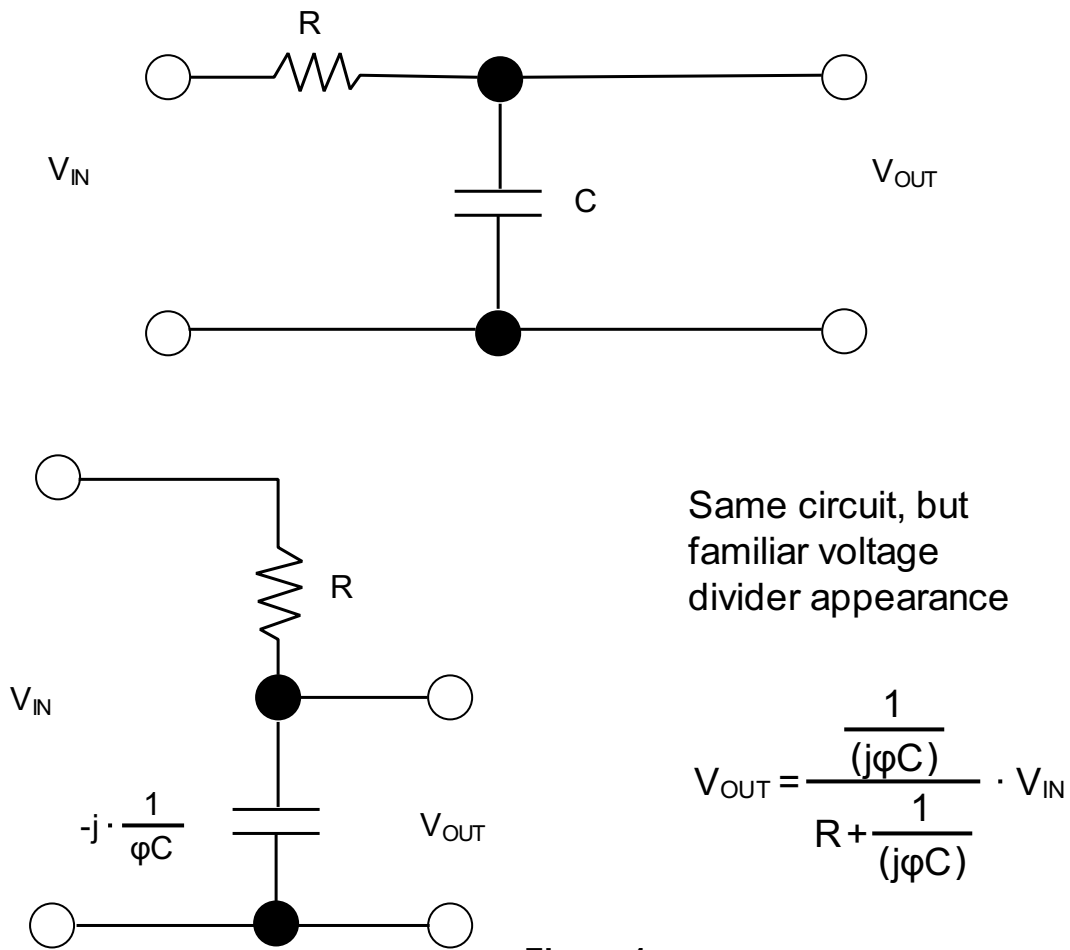


Figure 1.

Example:

To figure out how this network works, we find the transfer function. We begin by using the voltage divider to find V_{OUT} in the terms of V_{IN} , and consider there is no load (open output or $R_L = \infty$):

$$V_{OUT} = \frac{1}{1 + j\omega RC} \cdot V_{IN}$$

The transfer function is then found by rearranging the equation:

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega RC} \cdot V_{IN}$$

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\zeta\omega} \quad \zeta = RC$$

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + \frac{j\omega}{\omega_c}} \quad \omega_c = \frac{1}{RC}$$

The magnitude and phase of H_1 are:

$$|H_1| = \left| \frac{V_{OUT}}{V_{IN}} \right| = \frac{1}{\sqrt{1 + \zeta^2 \cdot \omega^2}}$$

$$\arg H = \theta = -\text{atan}\left(\frac{\omega}{\omega_c}\right)$$

Here, ζ is called the time constant, and ω_c is called the angular cutoff frequency of the circuit - related to the standard cutoff frequency by $\omega_c = 2\pi f_c$. The cutoff frequency represent the frequency at which the output voltage is attenuated by a factor of $1/\sqrt{2}$, the equivalent of half power. The cutoff frequency in this example is:

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi \cdot RC}$$

$$R := 50 \text{ Ohm}$$

$$C := 0.1 \cdot 10^{-6} \text{ F}$$

$$f_c := \frac{1}{2\pi \cdot RC}$$

$$f_c = 31830.989 \text{ s}^{-1}$$

Intuitively, we imagine that when the input voltage is very low in frequency, the capacitor's reactance is high, so it draws little current, thus keeping the output amplitude near the input amplitude. However, as the frequency of the input signal increases, the capacitor's reactance decreases and the capacitor draws more current, which in turn causes the output voltage to drop.

The capacitor produces a delay. At very low frequency, the output voltage follows the input - they have similar phases. As the frequency rises, the output starts lagging the input. At the cutoff frequency, the output voltage lags by 45° . As the frequency goes to infinity, the phase lag approaches 90° .

Figure 2 shows an RL low-pass filter that uses inductive reactance as the frequency-sensitive element instead of capacitive reactance, as was the case in the RC filter.

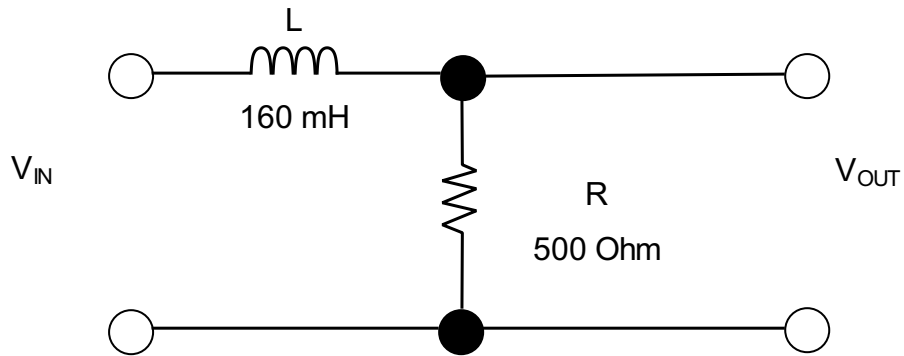
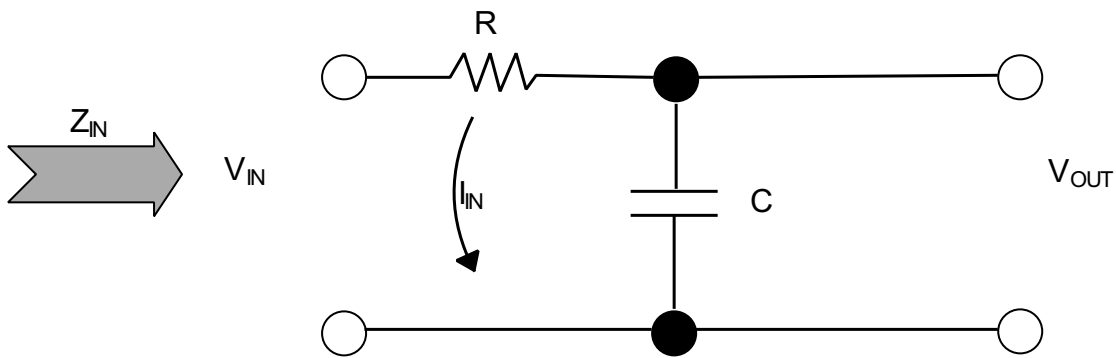


Figure 2.

The input impedance can be found by definition $Z_{IN} = V_{IN} / I_{IN}$, while the output impedance can be found by "killing the source" (Figure 3.)

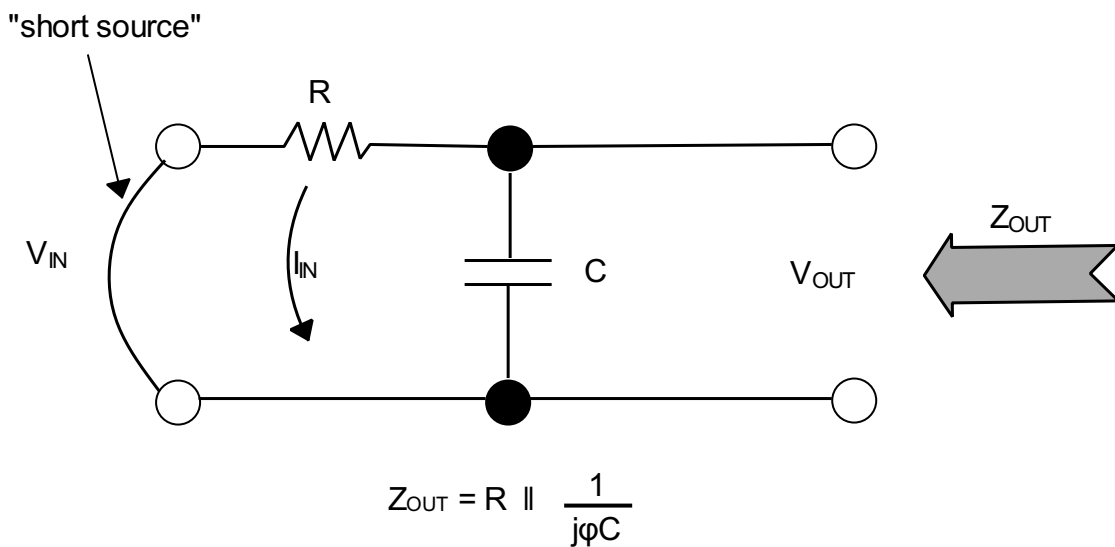
Input Impedance



$$Z_{IN} = \frac{V_{IN}}{I_{IN}}$$

$$Z_{IN} = R + \frac{1}{j\omega C}$$

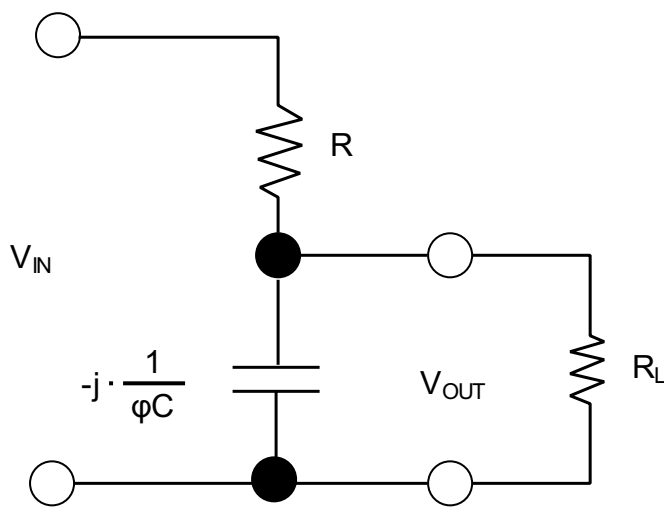
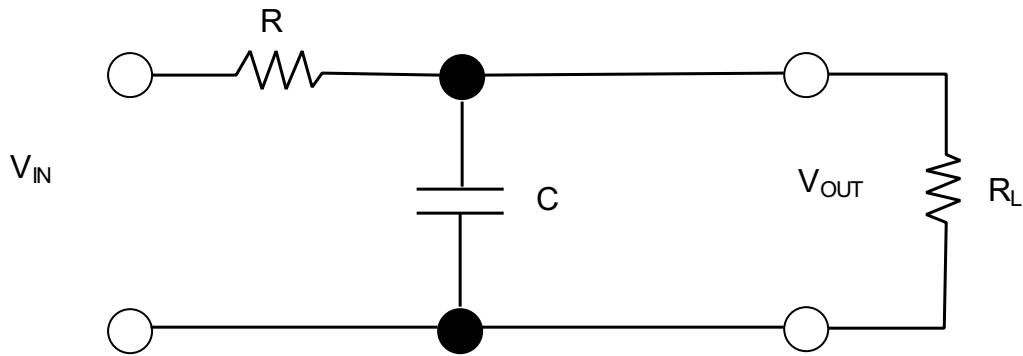
Output Impedance



$$Z_{OUT} = R \parallel \frac{1}{j\omega C}$$

Figure 3.

Now what happens when we put a finite load resistance R_L on the output? Doing the voltage divider stuff and preparing the voltage transfer function, we get:



Same circuit, but familiar voltage divider appearance

$$V_{OUT} = \frac{\frac{1}{j\phi C} \parallel R_L}{R + \left(\frac{1}{j\phi C} \parallel R_L\right)} \cdot V_{IN}$$

$$V_{OUT} = \frac{R'}{1 + j(\phi R' C)} \cdot V_{IN}$$

Figure 4.

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{\frac{1}{j\phi C} \parallel R_L}{R + \left(\frac{1}{j\phi C} \parallel R_L\right)} = \frac{R'}{1 + j \cdot (\phi R' C)}$$

where $R' = R \parallel R_L$

This is similar to the transfer function for the unterminated RC filter, but with resistance R being replaced by R' . Therefore:

$$\phi = \frac{1}{R' C} = \frac{1}{(R \parallel R_L) \cdot C_1} \quad \text{and} \quad H_1 = \frac{\frac{R'}{R}}{1 + j \left(\frac{\phi}{\phi_c}\right)}$$

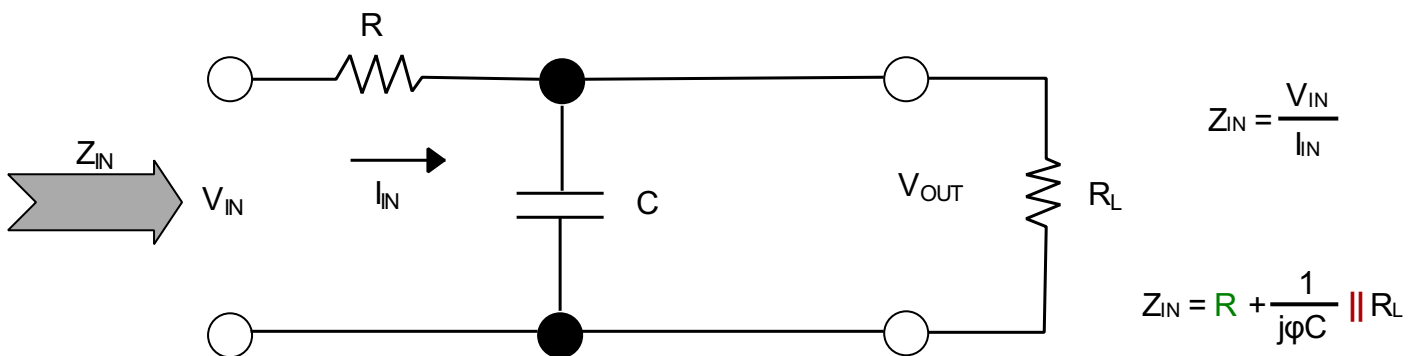
As you can see, the load has the effect of reducing the filter gain ($K = R' / R < 1$) and shifting the cutoff frequency to a higher frequency as ($R' = R \parallel R_L < R$).

The input and output impedance with load resistance become:

$$Z_{IN} = R + \frac{1}{j\omega C} \parallel R_L \quad \text{and} \quad Z_{IN|Min} = R$$

$$Z_{OUT} = R \parallel \frac{1}{j\omega C} \quad \text{and} \quad Z_{OUT|Max} = R$$

Input Impedance



Output Impedance

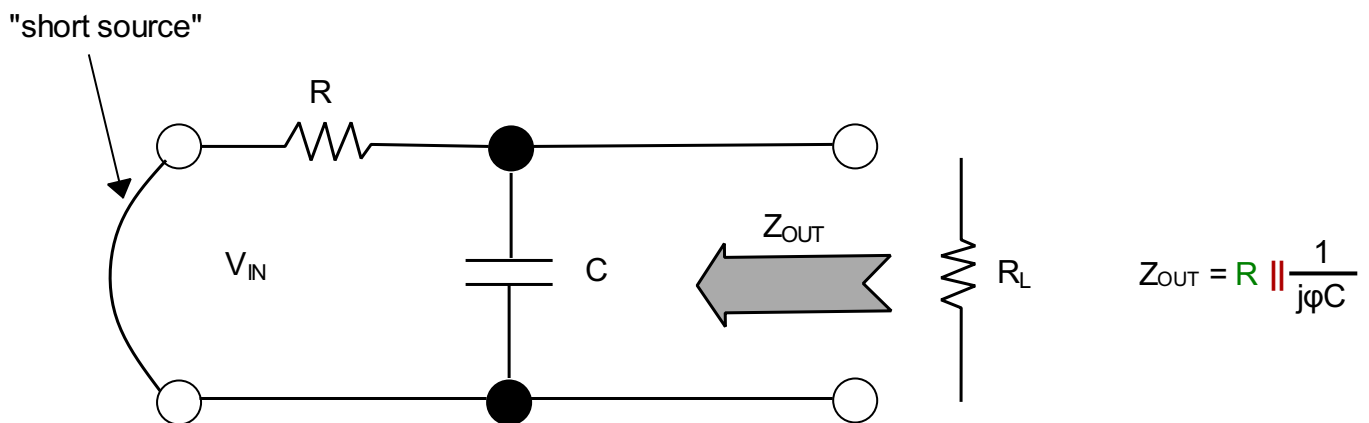


Figure 5.

As long as $R_L \gg Z_{OUT}$ or $R \gg Z_{OUT|Max} = R$ (condition for good voltage coupling), $R' \approx R$ and the terminated RC filter will look exactly like an unterminated filter. The filter gain is one, the shift in cutoff frequency disappears, and the input and output resistance become the same as before.

Example:

To find the transfer function or attenuation of the RL circuit with no load, we find V_{OUT} in terms of V_{IN} using the voltage divider:

$$V_{OUT} = \frac{R}{R + j\omega L} \cdot V_{IN} = \frac{1}{1 + j\left(\frac{\omega L}{R}\right)} \cdot V_{IN}$$

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\left(\frac{\omega L}{R}\right)}$$

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\left(\frac{\omega}{\omega_C}\right)}, \quad \omega_C = \frac{R}{L}$$

The magnitude and phase become:

$$|H_1| = \left| \frac{V_{out}}{V_{IN}} \right| = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}}$$

$$\arg H = \varphi = \text{atan}\left(\frac{\omega L}{R}\right) = \text{atan}\left(\frac{\omega}{\omega_C}\right)$$

Here, ω_C is called the angular cutoff frequency of the circuit - related to the standard cutoff frequency by $\omega_C = 2\pi f_C$. The cutoff frequency represents the frequency at which the output voltage is attenuated by a factor $1/\sqrt{2}$, the equivalent of half power. The cutoff frequency in this example is:

$$f_C = \frac{\omega_C}{2\pi} = \frac{R}{2\pi \cdot L}$$

$$R_1 := 500 \text{ Ohm} \quad L := 160 \cdot 10^{-3} \text{ H}$$

$$f_{C1} := \frac{R_1}{2\pi \cdot L}$$

$$f_{C1} = 497.359 \text{ s}^{-1}$$

Intuitively we imagine that when the input voltage is very low in frequency, the inductor doesn't have a hard time passing current to the output. However, as the frequency gets big, the inductor's reactance increases, and the signal becomes more attenuated at the output.

The inductor produces a delay. At very low frequency, the output voltage follows the input - they have similar phases. As the frequency rises, the output starts lagging the input. At the cutoff frequency, the

output voltage lags by 45° . As the frequency goes to infinity, the phase lag approaches 90° . Here ζ is called the time constant and φ_C is called the angular cutoff frequency of the circuit - related to the standard cutoff frequency by $\varphi_C = 2 \pi f_C$. The cutoff frequency represents the frequency at which the output voltage is attenuated by a factor of $1 / \sqrt{2}$, the equivalent of half power. The cutoff frequency in this example is:

The input impedance can be found using the definition of the input impedance:

$$Z_{IN} = \frac{V_{IN}}{I_{IN}} = j\varphi L + R$$

$$f_0 = \frac{\varphi_0}{2 \pi} = \frac{1}{2 \pi \cdot RC}$$

$$R_2 := 10000 \text{ Ohm}$$

$$C_2 := 0.1 \cdot 10^{-6} \text{ F}$$

$$f_0 := \frac{1}{2 \pi \cdot R_2 C_2}$$

$$f_0 = 159.155 \text{ s}^{-1}$$

The value of the input impedance depends on the frequency φ . For good voltage coupling, the input impedance of this filter should be much larger than the output impedance of the previous stage. The minimum value of Z_{IN} is an important number, and its value is minimum when the impedance of the inductor is zero ($\varphi \rightarrow 0$):

$$Z_{IN|Min} = R$$

Intuitively, we imagine that when the input voltage is very low in frequency, the capacitor's reactance is very high, and hardly any signal is passed to the output. However, as the frequency rises, the capacitor's reactance decreases, and there is little attenuation at the output.

In term of phase, at very low frequency the output leads the input in phase by 90° . As the frequency rises to the cutoff frequency, the output leads by 45° . When the frequency goes toward infinity, the phase approaches 0, the point where the capacitors acts like a short.

Input and output impedance of this filter can be found in a way similar to finding these impedance for low-pass filters: The output impedance can be found by sorting the source and finding the equivalent impedance between output terminals:

$$Z_{OUT} = j\varphi L \parallel R$$

where the source resistance is ignored. The output impedance also depends on the frequency φ . For good voltage couple, the output impedance of this filter should be much smaller than the input impedance of the next stage. The maximum value of Z_{OUT} is also an important number, and it is maximum when the impedance of the inductor is infinity ($\varphi \rightarrow \infty$)

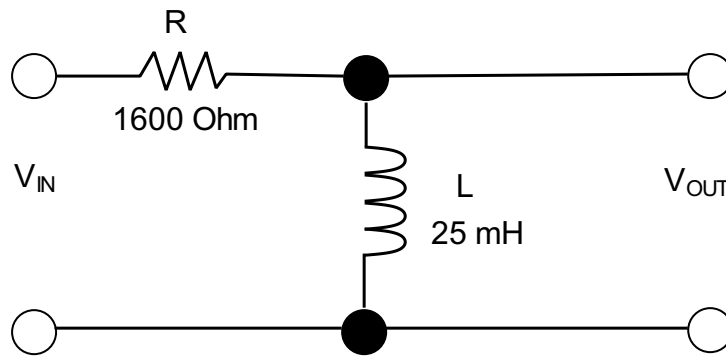


Figure 7.

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{j\varphi L}{R + j\varphi L} = \frac{\varphi L \angle 90^\circ}{\sqrt{R^2 + (\varphi L)^2} \cdot \text{atan}\left(\frac{\varphi L}{R}\right)}$$

$$H_1 = \frac{1}{1 - \frac{j\varphi C}{\varphi}}, \quad \varphi_C = \frac{R}{L}$$

$$Z_{IN} = R + \frac{1}{j\varphi C} \quad \text{and} \quad Z_{IN|Min} = R$$

$$Z_{OUT} = R \parallel \frac{1}{j\varphi C}, \quad Z_{OUT|Max} = R$$

$$Z_{OUT|Max} = R$$

When the RL low-pass filter is terminated with a load resistance R_L , the voltage transfer function changes to: With a terminated load resistance, the voltage transfer function becomes:

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{R \parallel R_L}{R \parallel R_L + \frac{1}{j\varphi C}} = \frac{1}{1 - j\left(\frac{1}{\varphi R' C}\right)} \quad \text{where} \quad R' = R \parallel R_L$$

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + \frac{j\varphi}{\varphi_C}} \quad \text{where} \quad \varphi_C = (R \parallel R_L) / L$$

$$f_c = \frac{\varphi_0}{2\pi} = \frac{R}{2\pi \cdot L}$$

$$R_3 := 1600 \text{ Ohm}$$

$$L_3 := 25 \cdot 10^{-3} \text{ H}$$

$$f_{c2} := \frac{R_3}{2\pi \cdot L_3}$$

$$f_{c2} = 10185.916 \text{ s}^{-1}$$

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{R}{R + \frac{1}{j\varphi C}} = \frac{1}{1 - j\left(\frac{1}{\varphi RC}\right)} = \frac{j\varphi\zeta}{1 + j\varphi\zeta}, \quad \zeta = RC$$

or

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{j\left(\frac{\varphi}{\varphi_C}\right)}{1 + j\left(\frac{\varphi}{\varphi_C}\right)} = \frac{1}{1 - \frac{j\varphi_C}{\varphi}}, \quad \varphi_C = \frac{1}{RC}$$

We imagine that when the input voltage is very low in frequency, the inductor's reactance is very low, so most of the current is diverted to ground - the signal is greatly attenuated at the output. However, as the frequency rises, the inductor's reactance increases and less current is passed to ground - the attenuation decreases.

In terms of phase, at very low frequency the output leads the input in phase by 90° . As the frequency rises to the cutoff frequency, the output leads by 45° . When the frequency goes toward infinity, the phase approaches 0, the point where the inductors acts like an open circuit.

The input and output impedances are:

As long as $R_L \gg Z_{OUT}$ or $R_L \gg Z_{OUT|Max} = R$ (condition for good voltage coupling), $R' = R$ and the terminated RC filter will look like an unterminated filter. The shift in cutoff frequency disappears, and input and output resistance become the same as before.

1.1.1.1.1. RL High-Pass Filter

In examples above we used RC filter with capacitive reactance, now we will show RL high-pass filter that uses inductive reactance as the frequency-sensitive element.

Example:

To find the transfer function or attenuation of the RL circuit, we again use the voltage divider equation and solve for the transfer function or attenuation of the RL circuit in terms of V_{OUT} and V_{IN} :

To figure out how this network works, we find the transfer function by using the voltage divider equation and solving in terms of V_{OUT} and V_{IN} :
 This is similar to the transfer function for the unterminated RS filter, but with resistance R being replaced by R' :

$$\arg H = \rho = 90^\circ - \operatorname{atan}\left(\frac{\varphi L}{R}\right)$$

$$\varphi_c = \frac{1}{R'C} = \frac{1}{(R \parallel R_L) \cdot C_1} \quad \text{and} \quad H_1 = \frac{1}{1 - \frac{j \cdot \varphi_c}{\varphi}}$$

RC High-Pass Filter Circuit

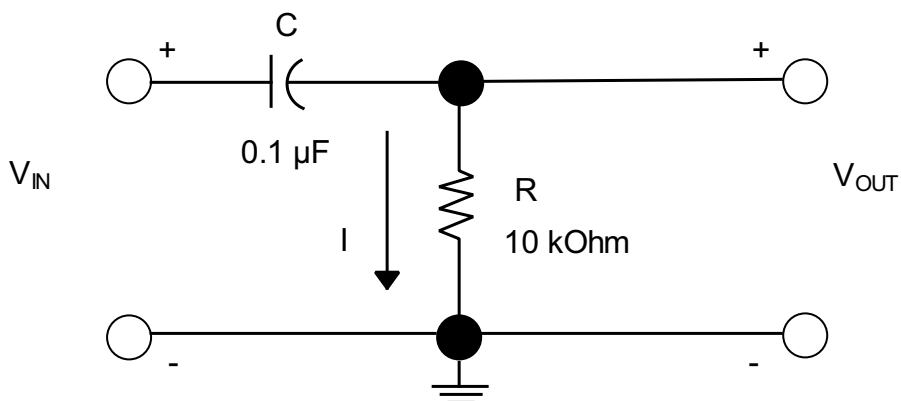


Figure 6.

The input impedance becomes:

$$Z_{IN} = j\varphi L = R \parallel R_L, \quad Z_{IN|Min} = R \parallel R_L$$

The magnitude and phase of H are:

$$|H_1| = \left| \frac{V_{OUT}}{V_{IN}} \right| = \frac{\zeta \varphi}{\sqrt{1 + \zeta^2 \cdot \varphi^2}}$$

$$\arg H = \rho = \operatorname{atan}\left(\frac{\varphi_c}{\varphi}\right)$$

Here ω_c is called the angular cutoff frequency of the circuit - related to the standard cutoff frequency by $\omega_c = 2\pi f_c$. The cutoff frequency represents the frequency at which the output voltage is attenuated by a factor of $1/\sqrt{2}$, the equivalent of half power. The cutoff frequency in this example is:
The output impedance becomes:

$$Z_{IN} = R + j\omega L \quad , \quad Z_{IN|Min} = R$$

$$Z_{OUT} = R \parallel j\omega L \quad , \quad Z_{OUT|Max} = R$$

$$Z_{OUT} = (j\omega L) \parallel R \quad , \quad Z_{OUT|Max} = R$$

The load has the effect of shifting the cutoff frequency to a higher frequency ($R' = R \parallel R_L < R$).
The output and input impedances are: The effect of the load is to shift the cutoff frequency to a lower value. Filter gain is not affected. Again, for

$R_L \gg Z_{OUT}$ or $R_L \gg Z_{OUT|Max} = R$ (condition for good voltage coupling), the shift in cutoff frequency disappears, and the filter will look exactly like an unterminated filter.

The magnitude and phase of H are:

For a terminated RL high-pass filter with load resistance, we do a similar calculation as we did with the RC high-pass filter, replacing the resistance with R':

1.1.1.1.2. High-Pass Filters

Example:

$$H_1 = \frac{V_{OUT}}{V_{IN}} = \frac{\frac{R'}{R}}{1 - \frac{j\omega L}{R}} \quad , \quad \omega_c = \frac{R'}{L} \quad , \quad R' = R \parallel R_L$$

$$|H_1| = \left| \frac{V_{OUT}}{V_{IN}} \right| = \frac{\omega L}{\sqrt{R^2 + (\omega L)^2}} = \frac{\frac{\omega}{\omega_c}}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

The input and output impedances are:

$$Z_{IN} = R + j\omega L \parallel R_L \quad , \quad Z_{IN|Min} = R$$

$$Z_{OUT} = R \parallel j\omega L \quad , \quad Z_{OUT|Max} = R$$

The load has the effect of lowering the gain, $K = R'/R < 1$, and it shifts the cutoff frequency to a lower value. As long as $R_L \gg Z_{OUT}$ or $R_L \gg Z_{OUT|Max} = R$ (condition for good voltage coupling), $R' \approx R$ and the terminated RC filter will look like an unterminated filter.

$$Z_{IN} = \frac{1}{j\omega C} + R \parallel R_L \quad , \quad Z_{IN|Min} = R \parallel R_L$$

$$Z_{OUT} = R \parallel \frac{1}{j\omega C} \quad , \quad Z_{OUT|Max} = R$$

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