Usage of exponential and logarithmic functions

These two groups of functions are very useful in real world situations. They are commonly used in many aspects of everyday life and especially in scientific research. The most common usage of exponential functions are: population growth and modeling, compounded interest calculations, exponential decay calculations, carbon dating, Newton's Law of Cooling, drug concentration,...
On the other hand, logarithmic functions are most commonly used for: measurement scales (Richter, Decibel, Google PageRank,..), interest rates, measurement of ionic compounds, astronomy, chemistry, ...

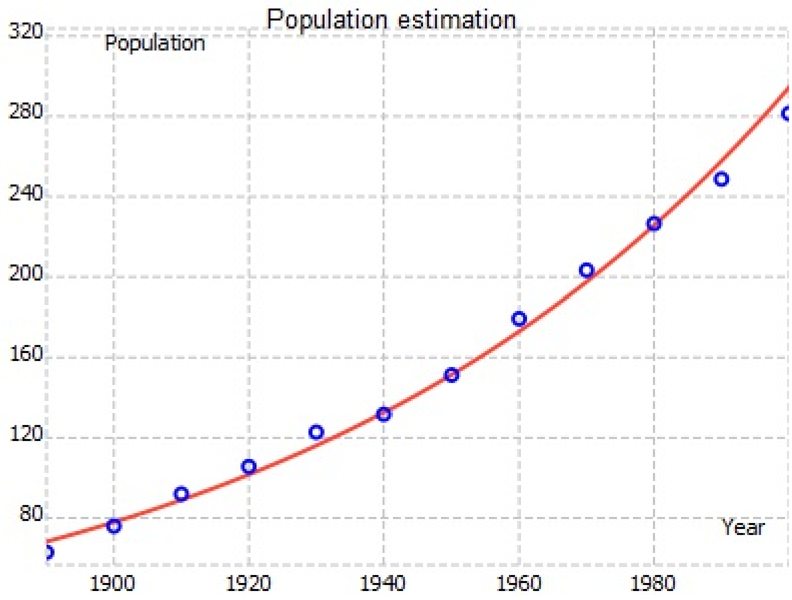
Exponential functions - Prediction

Use the data regarding the population of the US from the table below to predict the population for the year 2010. Then compare with the actual 2010 population which is approximately 308 million.

Year	Population (millions)
1890	62.9
1900	76
1910	92
1920	105.7
1930	122.8
1940	131.7
1950	151.3
1960	179.3
1970	203.3
1980	226.5
1990	248.7
2000	281.4

$$xu := \begin{bmatrix} 1890 & 1900 & 1910 & 1920 & 1930 & 1940 & 1950 & 1960 & 1970 & 1980 & 1990 & 2000 \\ 62.9 & 76 & 92 & 105.7 & 122.8 & 131.7 & 151.3 & 179.3 & 203.3 & 226.5 & 248.7 & 281.4 \end{bmatrix}$$

With this data, we can create a prediction function and visualize both the data and prediction function. The prediction function will be calculated using the MatDeck function **expfit** used for exponential regression. The resulting function is a standard exponential function in the form of $y(x) = ab^x$.



Exponential regression formula

$$\text{expfit}(xu) = 8.467e\text{-}10 \cdot e^{0.013\ x}$$

To predict the US population based on the exponential regression from the historical data, we will use x = 2010 in the regression formula.

$$\text{Prediction} := \text{replace symbols}(\text{expfit}(xu), x, 2010)$$

Prediction

The exponential model estimates that the 2010 population would be 336 million, an overestimate of approximately 28 million people.

Logarithmic function - Radioactive decay

One of the common applications of logarithms is in science, especially calculating the time it takes for half of the unstable material in a sample of radioactive substance to decay (this property is known as half-life). Knowing the half-life of the substance allows us to calculate the amount of the substance that is still radioactive after a specified time. We can use the formula for radioactive decay:

$$A(t) = A_0 \cdot e^{\frac{\ln(0.5)}{T} \cdot t}$$

A_0 - the initial amount of substance
T - half-life of the substance
t - time period
y or A(t) - the amount of substance after time t

The following table lists the half-life for several of the common radioactive substances.

Substance	Use	Half-life
Gallium-67	Nuclear medicine	80 hours
Cobalt-60	Manufacturing	5.3 years
Technetium-99m	Nuclear medicine	6 hours
Americium-241	Construction	432 years
Carbon-14	Archeological dating	5715 years
Uranium-235	Atomic power	703800000years

Now, we can calculate how long it would take for twenty percent of a 500-gram sample of uranium-235 to decay.

$$y = A_0 \cdot e^{\frac{\ln(0.5)}{T} \cdot t}$$ Radioactive decay formula

$$400 = 500 \cdot e^{\frac{\ln(0.5)}{703800000} \cdot t}$$ After 20% decays, 400 grams are left

$$400 == 500 \cdot e^{\frac{\ln(0.5)}{703800000} \cdot t} = \left[t \quad 226572993.182 \right]$$ MatDeck equation solving

It would take 226.572.993 years for 20% of the 500-gram sample to decay .