Linear equations and systems in MatDeck

Linear equations are equations that can be put in following form

$$a_1 \cdot X_1 + a_2 \cdot X_2 + ... + a_n \cdot X_n + b = 0$$

In this form of the equation, $X_1, X_2, ..., X_n$ are the variables and $a_1, a_2, ..., a_n$, b are the coefficients. Coefficients can be real or imaginary numbers, and at least one of the equation coefficients have to be nonzero if we want to have a meaningful equation.

In MatDeck, we have implemented a linear equation solver engine and function for solving linear equations as well. To solve a linear equation, insert the equation object from the Math tab, define the equation and place the equal sign after the equation.

$$\frac{x+3}{2} - \frac{x-1}{4} == -1 = \begin{bmatrix} x & -11 \end{bmatrix}$$

As you can see, the result is a vector whose first element presents the symbol for which we have solved an equation, and the second element of the resulting vector is the solution of inputted equation.
If we create an equation of two or more variables, the result will be a matrix of solutions where every row will present a solution for one of the variables

$$\frac{x+3}{2} - \frac{y-1}{4} == -1 = \begin{bmatrix} x & 0.5\ y - 5.5 \\ y & 2\ x + 11 \end{bmatrix}$$

The alternative way of solving linear equations is to use the function **linsolve**. The function **linsolve** will return only the result of the equation because the second argument in the function is the symbol for which we are solving the equation.

$$\text{linsolve}\left(\frac{x+3}{2} - \frac{x-1}{4} == -1\ ,\ x\right) = -11$$

$$\text{linsolve}\left(\frac{x+3}{2} - \frac{y-1}{4} == -1\ ,\ y\right) = 2\ x + 11$$

The two ways of presenting and solving linear equations that we have mentioned above, can be used to solve equations in all the different forms that they can be written.

<u>Slope-intercept form</u> in which we write the equation in form of y = f(x) is supported.

$$y == -\frac{4}{3} \cdot x + \frac{3}{7} = \begin{bmatrix} y & -1.333\ x + 0.429 \\ x & -0.75\ y + 0.321 \end{bmatrix}$$

$$\text{linsolve}\left(y == -\frac{4}{3} \cdot x + \frac{3}{7}, x\right) = -0.75\ y + 0.321$$

Point-slope form of equation writing is also supported and will be calculated

$$y-5 == 0.84 \cdot (x+2) = \begin{bmatrix} y & 0.84\ x+6.68 \\ x & 1.190\ y-7.952 \end{bmatrix}$$

$$\text{linsolve}(y-5 == 0.84 \cdot (x+2),\ y) = 0.84\ x+6.68$$

Two-point form where the equation pass through points (x_1,y_1) and (x_2,y_2).

$$y+2.45 == \frac{-3+2.45}{7-3.1} \cdot (x-3.1) = \begin{bmatrix} y & -0.141\ x-2.013 \\ x & -7.091\ y-14.273 \end{bmatrix}$$

$$\text{linsolve}\left(y+2.45 == \frac{-3+2.45}{7-3.1} \cdot (x-3.1),\ x\right) = -7.091\ y-14.273$$

Symmetric form is very similar to two-point form where both sides are multiplied by same factor.

$$(7-3.1) \cdot (y+2.45) == (-3+2.45) \cdot (x-3.1) = \begin{bmatrix} y & -0.141\ x-2.013 \\ x & -7.091\ y-14.273 \end{bmatrix}$$

$$\text{linsolve}((7-3.1) \cdot (y+2.45) == (-3+2.45) \cdot (x-3.1),\ x) = -7.091\ y-14.273$$

Intercept form is also supported as linear equation solver

$$\frac{x}{4} + \frac{y}{-2} == 1 = \begin{bmatrix} x & 2\ y+4 \\ y & 0.5\ x-2 \end{bmatrix}$$

$$\text{linsolve}\left(\frac{x}{4} + \frac{y}{-2} == 1,\ x\right) = 2\ y+4$$

Determinant form

$$\det\left(\begin{bmatrix} x-3.1 & y+2.45 \\ 7-3.1 & -3+2.45 \end{bmatrix}\right) == 0 = \begin{bmatrix} x & -7.091\ y-14.273 \\ y & -0.141\ x-2.013 \end{bmatrix}$$

$$\text{linsolve}\left(\det\left(\begin{bmatrix} x-3.1 & y+2.45 \\ 7-3.1 & -3+2.45 \end{bmatrix}\right) == 0,\ x\right) = -7.091\ y-14.273$$

$$
\begin{aligned}
&a_{11} \cdot x_1 + a_{12} \cdot x_2 + ... + a_{1n} \cdot x_n = b_1 \\
&a_{21} \cdot x_1 + a_{22} \cdot x_2 + ... + a_{2n} \cdot x_n = b_2 \\
&.... \\
&a_{n1} \cdot x_1 + a_{n2} \cdot x_2 + ... + a_{nn} \cdot x_n = b_n
\end{aligned}
$$

If $b_1 = b_2 = ... = b_n$, then for the above linear system it can be said that it is a homologous system. Ordered n numbers (ε_1, ε_2,..., ε_n) are the solutions of the linear equation system if every equation in system for $x_k = \varepsilon_k$ (k = 1, 2,..., n) comes down to a identity. For systems that satisfy this condition, we can say it is a solvable system.

The general form of a linear equation system can be presented in several different ways. One of them is Vector equation form.

$$
x_1 \cdot \begin{bmatrix} a_{11} \\ a_{21} \\ ... \\ a_{n1} \end{bmatrix} + x_2 \cdot \begin{bmatrix} a_{12} \\ a_{22} \\ ... \\ a_{n2} \end{bmatrix} + ... + x_n \cdot \begin{bmatrix} a_{1n} \\ a_{2n} \\ ... \\ a_{nn} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ ... \\ b_n \end{bmatrix}
$$

Another common form of writing systems of linear equations is Matrix equation form.

$$
\begin{bmatrix} a_{11} & a_{12} & ... & a_{1n} \\ a_{21} & a_{22} & ... & a_{2n} \\ ... & ... & ... & ... \\ a_{m1} & a_{m2} & ... & a_{mn} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ ... \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ ... \\ b_m \end{bmatrix}
$$

A - m x n matrix   X - column vector   b - column vector

In MatDeck we have implemented the functions **linsolvesys** - Linear system equation solver and **nonlinsolvesys** - Nonlinear system equation solver. Both of them are capable of solving solvable systems, linear systems are solved in matrix forms and nonlinear systems are solved using numerical methods.

To solve linear equation systems using the linsolvesys function, insert the vector as the first function argument. Place the Equation object from Math tab as the vector elements and start inserting the equations. We will show you several examples of linear equations system solving.

$$\mathrm{linsolvesys}\left(\begin{bmatrix}2\,x+3\,y-5\,z==-7\\-3\,x+2\,y+z==-9\\4\,x-y+2\,z==17\end{bmatrix},\begin{bmatrix}x\\y\\z\end{bmatrix}\right)=\begin{bmatrix}3\\-1\\2\end{bmatrix}$$

$$\mathrm{linsolvesys}\left(\begin{bmatrix}2\,x+3\,y-5\,z==0\\-3\,x+2\,y+z==0\\4\,x-y+2\,z==0\end{bmatrix},\begin{bmatrix}x\\y\\z\end{bmatrix}\right)=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\mathrm{linsolvesys}\left(\begin{bmatrix}x1+x2+x3==3\\x2+x3+x4==4\\x3+x4+x5==5\\x4+x5+x6==6\\x5+x6+x7==7\\x6+x7+x1==8\\x7+x1+x2==9\end{bmatrix},\begin{bmatrix}x1\\x2\\x3\\x4\\x5\\x6\\x7\end{bmatrix}\right)=\begin{bmatrix}3\\1\\-1\\4\\2\\3.331e\text{-}16\\5\end{bmatrix}$$

Syntax for using the function nonlinsolvesys is the same as for linsolvesys, and the differences are that you can use nonlinear equations in a system and you have to define the starting point for variables. If the system has several solutions, this starting point will localize the solution and the function will return the nearest solution. Function nonlinsolvesys can calculate only real solutions.

$$\mathrm{nonlinsolvesys}\left(\begin{bmatrix}\sqrt[3]{x\cdot y}+\sqrt[2]{z}==4\\x^2+y^2+z^2==81\\\sqrt[2]{x}+y\cdot z==33\end{bmatrix},\begin{bmatrix}x\\y\\z\end{bmatrix},\begin{bmatrix}2\\10\\5\end{bmatrix}\right)=\begin{bmatrix}1\\8\\4\end{bmatrix}$$

To illustrate how the inserted starting point affects the final solution of the function, take a look at the following example.

$$\text{nonlinsolvesys}\left(\begin{bmatrix} x^4 + y^2 == 17 \\ x^2 + y^2 == 5 \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\text{nonlinsolvesys}\left(\begin{bmatrix} x^4 + y^2 == 17 \\ x^2 + y^2 == 5 \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\text{nonlinsolvesys}\left(\begin{bmatrix} x^4 + y^2 == 17 \\ x^2 + y^2 == 5 \end{bmatrix}, \begin{bmatrix} x \\ y \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

As you can see, for three different starting points we have calculated three different solutions of the inserted system.