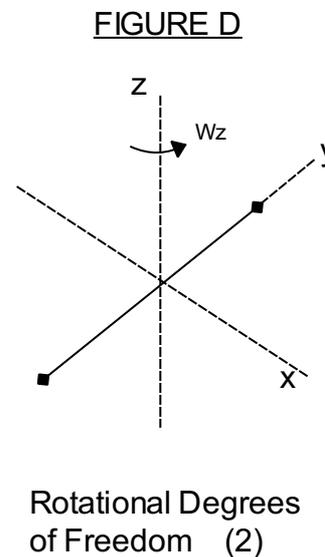
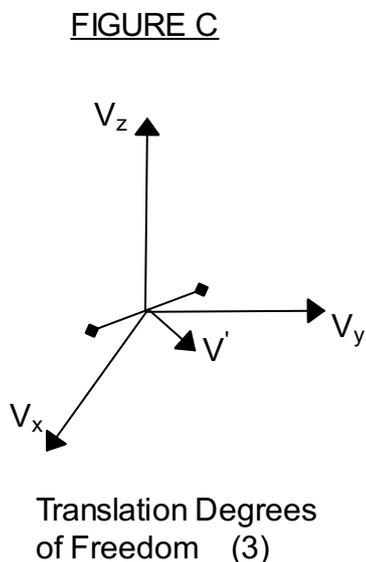
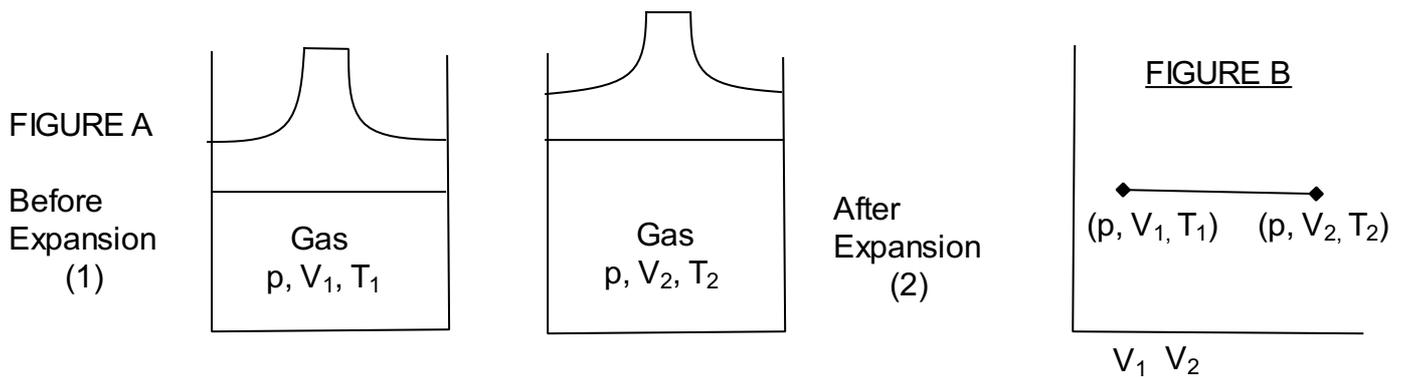


Gas equations

Three moles of a diatomic, perfect gas is allowed to expand at constant pressure. The initial volume is 1.3 m^3 and the initial temperature is 350°K . If 10,000 Joules are transferred to the gas at the source of the heat, what is the final volume and temperature?



Solution

Since the process of the heat (Q) addition occurs at a constant pressure, we can write

$$Q := \mu C_p (T_2 - T_1)$$

Where μ is the number of moles of gas in the system, C_p is the molar specific heat at constant pressure and $T_2 - T_1$ is the temperature difference between the 2 equilibrium states (see figures (a) and (b)). Q is given (10,000 Joules), as is μ and T . We can solve T_2 as a function of C_p . If we can calculate the value of C_p by using kinetic theory, we would have obtained T_2 .

Consider a gas moving from an equilibrium state to another, via a thermodynamic process. We assume that the process occurs at a constant volume. Using the 1st Law of Thermodynamics, we obtain

$$\Delta U := Q - W \quad (1)$$

Here, ΔU is the change in the internal energy of the gas during the process and Q and W are the heat added to and work done by the gas, respectively, during the process. Writing (1) in differential form

$$dU = dQ - dW \quad (2)$$

In general, the element of work done by the gas in an expansion is, by definition

$$dW := F ds \quad (3)$$

Where F is the force the gas exerts on the piston (see figure (a)) and ds is the distance the piston moves during the expansion. Since F acts as the perpendicular to the face of the piston (F and ds are parallel).

But F can be written in terms of the pressure (p), that the gas exerts on the piston face of area (A). Using that in (3)

$$\begin{aligned} F &:= pA \\ dW &= p ds A = p dV \end{aligned}$$

Where dV is the differential change in volume of the gas during the expansion. Substituting this into (2), we find

$$dU := dQ - p dV \quad (4)$$

Applying this equation to the above-mentioned isovolumic process, we obtain $dU = dQ$ since $dV = 0$. By definition

$$dQ := \mu C_v dT$$

Where C_v is the molar specific heat of the gas at a constant volume and dT is the differential temperature change the gas experiences due to the addition of heat dQ .

$$dU := \mu C_v dT \quad (5)$$

We now assume that the change in internal energy of a gas is only a function of the temperature difference experienced by the gas. Then, no matter what thermodynamic process the gas experiences, (5) stays the same.

Consider next an isobaric thermodynamic process. Again, we apply (4). Since the process occurs at a constant pressure

$$dQ := \mu C_p dT \quad (6)$$

Where C_p is the molar heat capacity at a constant pressure. Furthermore, from the ideal gas law $pV = \mu RT$. If p is constant,

$$p \cdot \frac{dV}{dT} = \mu R$$

$$p dV = \mu R dT \quad (7)$$

Using (6), (7) and (5) in (4)

$$\mu C_v dT = \mu C_p dT - \mu R dT$$

$$C_p := C_v + R \quad (8)$$

Equation (8) relates the molar specific heat at a constant pressure to the molar specific heat at a constant volume.

All the derivations up to now have been necessary in order to obtain the certain relations involving molar specific heats, namely equations (5) and (8). We now turn to an examination of the internal energy of a diatomic gas. Each method of energy storage of a diatomic molecule is called a degree of freedom. If we view diatomic molecules as being dumbbell-shaped, then it was within 5 degrees of freedom (see figure (c) and (d)). The molecule may move translationally in 3 directions (x, y, z) with 3 kinetic energies ($\frac{1}{2} m v_x^2$, $\frac{1}{2} m v_y^2$, $\frac{1}{2} m v_z^2$). Furthermore, it may rotate about 3 axis (x, y, z), with kinetic energies ($\frac{1}{2} I_x \psi_x^2$, $\frac{1}{2} I_y \psi_y^2$, $\frac{1}{2} I_z \psi_z^2$). The rotational kinetic energy about the y axis is negligible (see figure (d)) because $I_y \ll I_x, I_z$. Hence a diatomic molecule has 5 independent methods of energy absorption, of 5 degrees of freedom. Each of these kinetic energy terms has the same form, mathematically. That is, they are all of the form of a positive constant times the square of a variable which has a domain extending from $-\infty$ to $+\infty$. The theorem of equipartition of energy tells us that, when Newtonian mechanics is present and the number of gas particles is large, each of these forms have the same average value per molecule ($\frac{1}{2} kT$). In other words, each degree of freedom of a gas molecule, contributes an amount of $\frac{1}{2} kT$, to the internal energy of the gas. For a diatomic gas then, the internal energy per molecule is

$$U_i := \frac{5}{2} \cdot kT$$

The internal energy for μ moles of the molecules is

$$U := \mu N_0 U_i$$

$$U = 2.5 kT N_0 \mu$$

Using this in equation (5), we can solve to find C_V

$$C_V := \frac{1}{\mu} \cdot \frac{dU}{dT}$$
$$C_V := 2.5 R$$

Using this fact and putting it into (8)

$$C_p := C_V + R$$
$$C_p = 3.5 R$$

Getting back to the original problem, use this value of C_p in the first equation

$$Q := \mu C_p (T_2 - T_1)$$
$$Q = 3.5 T_2 R \mu - 3.5 T_1 R \mu$$

$$T_1 := 350 \text{ K} \quad Q := 10000 \text{ J} \quad \mu := 3 \text{ mol} \quad R := 8.31 \frac{\text{J}}{\text{mol K}} \quad V_1 := 1.3 \text{ m}^3$$

$$T_2 := T_1 + \frac{Q}{3.5 \mu R}$$
$$T_2 = 464.606 \text{ K}$$

Using the ideal gas law for the 2 equilibrium states

$$pV_1 = \mu R T_1$$

$$pV_2 = \mu R T_2$$

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$V_2 := \frac{T_2}{T_1} \cdot V_1$$

$$V_2 = 1.726 \text{ m}^3$$