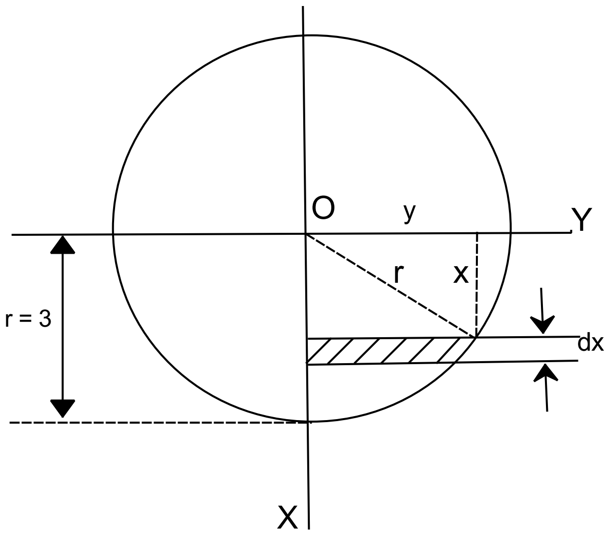
Hydrostatics 1

Calculate the total force on a gate closing a circular pipe 6 feet in diameter when the pipe is half full.



Solution:

y · dx is the differential of area.
xw is the fluid pressure which is depth x weight of 1 cubic feet of fluid, or density, w
So, **xw · y · dx** is the force on the differential area.

Summing up the differential fluid forces, where w = 62.5 lb./cu.ft. for water, we have R = w ∫ y · x · dx.

$$y := \sqrt[2]{r^2 - x^2}$$

Total force on one-half of the area of the water is

$$w := 62.5 \qquad r := 3$$

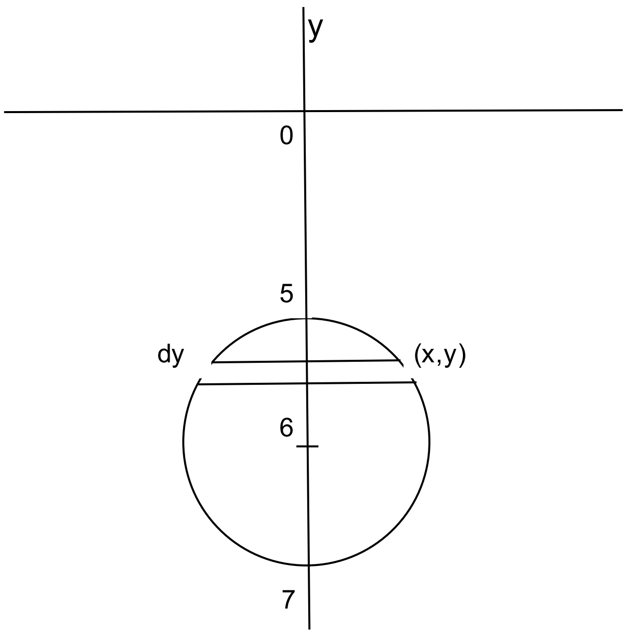
$$R := w \int_0^3 \sqrt[2]{r^2 - x^2} \cdot x \, dx$$

$$R = 562.501$$

Finally the total force on a gate for one half of the area of water is 562.5 lb.

Hydrostatics 2

The center of a circular floodgate of radius 2' in a reservoir is at a depth of 6' below the water surface. Find the total force on the gate if the water temperature is 40 fahrenheit.



Solution

Choose the axis with the origin at the center of the circle as shown in the figure. The equation of the circle is: $x^2 + y^2 = 4$. The area of the rectangular strip is: $dA = 2x\,dy$. y is in the area [-2,2]. Pressure on the strip, $p = w \cdot h$, where $w = 1000$ kg/m^3 ,is the density of water with the specified temperature, and $h = 6 - y$ is the depth of the strip below the water surface. The force on the strip: $dF = p\,dA = wh \cdot 2x\,dy$.

The total force on the gate is:

$$w := 1000 \qquad \text{kg/m}^3 \text{ - Water density}$$

$$x := \sqrt[2]{4 - y^2}$$

$$R := \int_{-2}^{2} w\,(6 - y) \cdot 2\,x\,dy$$

$$R = 75398.371$$

Finally, the total force on the gate will be 75398 kg/m^3.