Trigonometry - Number of solutions

In this example we will solve the problem of finding the number of solutions in the following equation

$$x = 4\pi \cdot \sin(x) \qquad (1)$$

We can divide the equation with 4π and transform it to the following form

$$\frac{x}{4\pi} = \sin(x)$$

As sin(x) can take a value from the interval [-1 , 1], we have

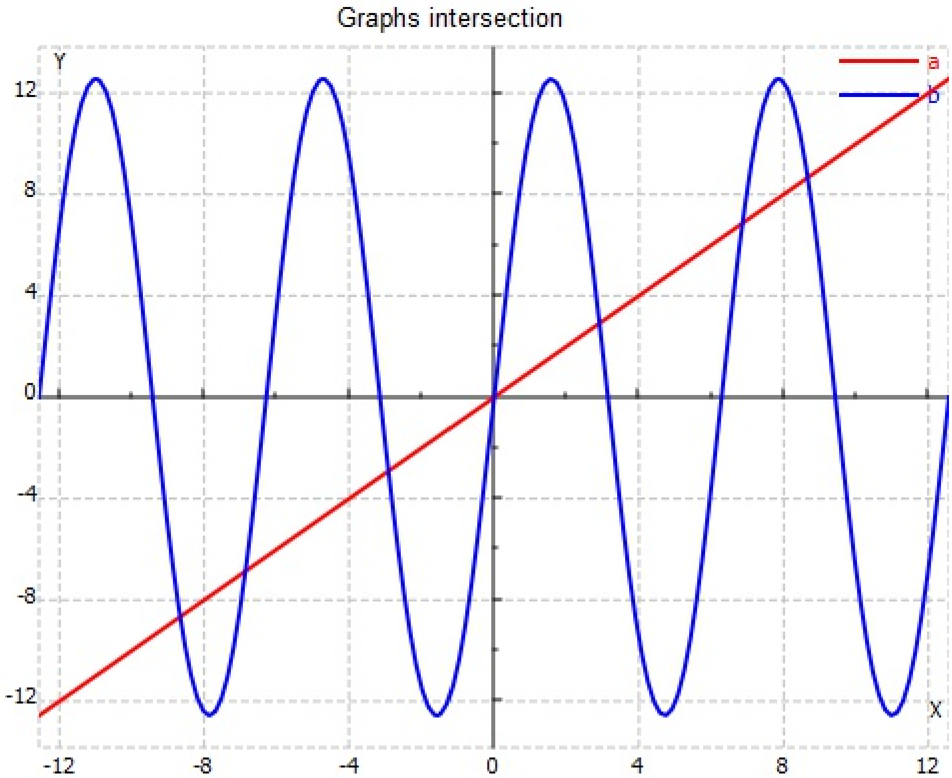
$$-1 \le \frac{x}{4\pi} \le 1$$

$$-4\pi \le x \le 4\pi$$

Now, we will plot every side of equation (1) as an independent graph and find the number of intersections between them. This number of intersections will be the number of solutions for equation (1). The variable x will take values from the interval [-4π , 4π], and that will be the border values in graph.

$$a := \text{curve2d}(x, -4\pi, 4\pi, 200)$$

$$b := \text{curve2d}(4 \cdot \pi \cdot \sin(x), -4\pi, 4\pi, 300)$$



The number of intersections on the graph is seven, so the equation have seven solutions.

To find the exact numerical values of those seven solutions we use the function nonlinsolve(). This is a non linear equation solver function. It's first argument is an equation object and the second is the symbol for which we are solving the specified equation

$$\text{nonlinsolve}\left(x == 4\ \pi \cdot \sin(x)\ ,\ x\right) = \begin{bmatrix} -8.664 & -6.861 & -2.908 & 0 & 2.908 & 6.861 & 8.664 \end{bmatrix}$$

As you can see, there are seven values, the same number of solutions we have counted from the intersections on graph. Now, we know their exact numerical values.

Solutions of this equation can be calculated in another way. Just create the equation object, define the equation and place the equal sign right from equation.

$$x == 4\ \pi \cdot \sin(x) = \begin{bmatrix} x & \begin{bmatrix} -8.664 & -6.861 & -2.908 & 0 & 2.908 & 6.861 & 8.664 \end{bmatrix} \end{bmatrix}$$

In the resulting vector, the first value is the symbol for which we are solving equation for and the second value is the vector of the solutions, the same one we had as a result of the nonlinsolve function.