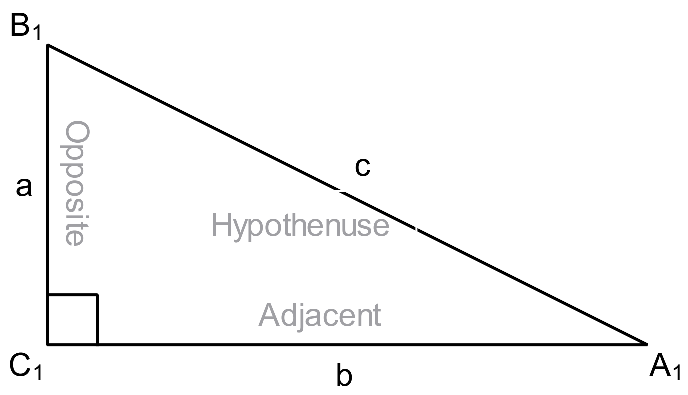
Trigonometry - basics

Trigonometry is a part of mathematics where the relationship between lengths and angles of triangles is studied. If we have a right angle triangle (a triangle in which one of the angles is 90 degrees) we can define the trigonometric values of one of the angles, one of the angles which is not 90 degrees, simply by using the ratios of the triangle sides. Let's look at the triangle on picture below and define the names of it's sides and angles



$$\sin(A_1) = \frac{\text{opposite}}{\text{hypothenuse}} = \frac{a}{c}$$

$$\cos(A_1) = \frac{\text{adjacent}}{\text{hypothenuse}} = \frac{b}{c}$$

$$\tan(A_1) = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b}$$

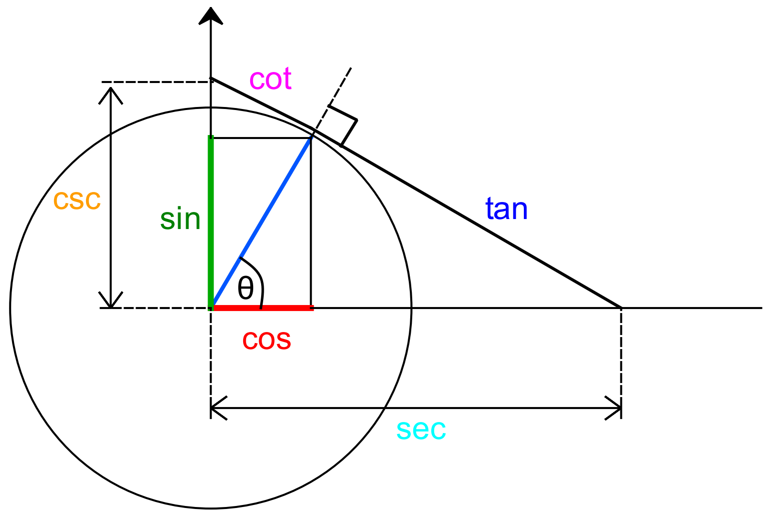
$$\cot(A_1) = \frac{\text{adjacent}}{\text{opposite}} = \frac{b}{a} = \frac{1}{\tan(A_1)}$$

$$\sec(A_1) = \frac{1}{\cos(A_1)} = \frac{\text{hypothenuse}}{\text{adjacent}} = \frac{c}{b}$$

$$\csc(A_1) = \frac{1}{\sin(A_1)} = \frac{\text{hypothenuse}}{\text{opposite}} = \frac{c}{a}$$

The inverse of the trigonometric functions defined above are: arcsine, arccosine, arctangent, arccotangent, arcsecant and arccosecant.

The functions we have defined only apply to angles up to 90 degrees. We can extend them to any positive or negative value using the unit circle. All trigonometric functions of an angle θ can be constructed geometrically in a unit circle.



Parity of trigonometric functions

$$\sin(-\alpha) = -\sin(\alpha) \qquad \cos(-\alpha) = \cos(\alpha) \qquad \tan(-\alpha) = -\tan(-\alpha) \qquad \cot(-\alpha) = -\cot(\alpha)$$

Pythagorean identities

$$\sin(\alpha)^2 + \cos(\alpha)^2 = 1 \qquad \tan(\alpha)^2 + 1 = \sec(\alpha)^2 \qquad \cot(\alpha)^2 + 1 = \csc(\alpha)^2$$

Euler's formula consequences

$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i} \qquad \cos(x) = \frac{e^{ix} + e^{-ix}}{2} \qquad \tan(x) = \frac{i \cdot \left(e^{-ix} - e^{ix}\right)}{e^{ix} + e^{-ix}}$$