Vectors and matrix - elementary operations and functions

In this document we will present and describe how the majority of vectors and matrix functions can be used to determine their properties and relationships. Also, we will describe the elementary operation with vectors and matrices and some expansions of this operations that are implemented in MatDeck.

Vector elementary operations

$$k \cdot \begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} k\,a & k\,b & k\,c \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \end{bmatrix} + \begin{bmatrix} d & e & f \end{bmatrix} = \begin{bmatrix} a+d & b+e & c+f \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \end{bmatrix} - \begin{bmatrix} c & d & e \end{bmatrix} = \begin{bmatrix} a-c & b-d & c-e \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \end{bmatrix} \cdot \begin{bmatrix} c \\ d \\ e \end{bmatrix} = c\,a + d\,b + e\,c$$

Vectors elementary operations

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \cdot \begin{bmatrix} c & d & e \end{bmatrix} = \begin{bmatrix} c\,a & d\,a & e\,a \\ c\,b & d\,b & e\,b \\ c^2 & d\,c & e\,c \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \end{bmatrix}^2 = \begin{bmatrix} a^2 & b^2 & c^2 \end{bmatrix}$$

MatDeck vectors properties

Matrix elementary operations

$$k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} k\,a & k\,b \\ k\,c & k\,d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} a1 & b1 \\ c1 & d1 \end{bmatrix} = \begin{bmatrix} a+a1 & b+b1 \\ c+c1 & d+d1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a1 & b1 \\ c1 & d1 \end{bmatrix} = \begin{bmatrix} a-a1 & b-b1 \\ c-c1 & d-d1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} a1 & b1 \\ c1 & d1 \end{bmatrix} = \begin{bmatrix} a1\,a + c1\,b & b1\,a + d1\,b \\ a1\,c + c1\,d & b1\,c + d1\,d \end{bmatrix}$$

Matrix elementary operations

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e \\ f \end{bmatrix} = \begin{bmatrix} e\,a+f\,b \\ e\,c+f\,d \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} \cdot \begin{bmatrix} c & d \\ e & f \end{bmatrix} = \begin{bmatrix} a\,c+b\,d \\ a\,e+b\,f \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 = \begin{bmatrix} a^2 & b^2 \\ c^2 & d^2 \end{bmatrix}$$

MatDeck matrix properties

Next, we move on to vectors and matrix functions that are supported in MatDeck

$$\mathrm{adj}\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Matrix adjoint

$$\mathrm{mat\ cofactor}\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$$

Matrix cofactor

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Matrix transpose

$$\begin{bmatrix} 1 & 2i \\ 3 & 4 \end{bmatrix}^* = \begin{bmatrix} 1+0i & 3+0i \\ 0-2i & 4+0i \end{bmatrix}$$

Matrix conjugatetranspose

$$\det\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = -2$$

Matrix determinant

$$\mathrm{diag}\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 4 \end{bmatrix}$$

Matrix diagonal

$$\mathrm{eigenvectors}\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = \begin{bmatrix} \begin{bmatrix} -0.825 \\ 0.566 \end{bmatrix} & \begin{bmatrix} -0.416 \\ -0.909 \end{bmatrix} \end{bmatrix}$$

Matrix eigenvectors

$$\mathrm{mat\ inverse}\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\right) = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

Matrix inverse

$$\text{is hermitian}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=\text{false}$$

Is matrix Hermitian

$$\text{matrref}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Matrix reduce echelon form

$$\text{mat mul}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=24$$

Matrix elements multiplication

$$\text{row mul}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=\begin{bmatrix}2\\12\end{bmatrix}$$

Matrix row elements multiplication

$$\text{col mul}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=\begin{bmatrix}3 & 8\end{bmatrix}$$

Matrix column elements multiplication

$$\text{negativedefinite}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=\text{false}$$

Is matrix negativedefinite

$$\text{negativesemidefinite}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=\text{false}$$

Is matrix negativesemidefinite

$$\text{positivedefinite}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=\text{false}$$

Is matrix positivedefinite

$$\text{positivesemidefinite}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=\text{false}$$

Is matrix positivesemidefinite

$$\text{rank}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=2$$

Matrix rank

$$\text{mat sum}\left(\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\right)=10$$

Matrix elements sum

$$\text{row sum}\left(\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\right)=\begin{bmatrix}3\\ 7\end{bmatrix}$$

Matrix rows elements sum

$$\text{col sum}\left(\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\right)=\begin{bmatrix}4 & 6\end{bmatrix}$$

Matrix columns elements sum

$$\text{elementssum}\left(\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, \text{"row"}\right)=\begin{bmatrix}3\\ 7\end{bmatrix}$$

Matrix rows/columns elements sum

$$\text{tr}\left(\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\right)=5$$

Matrix trace

$$\text{triangular}\left(\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, \text{"upp"}\right)=\begin{bmatrix}1 & 2\\ 0 & 4\end{bmatrix}$$

Create triangular matrix